Safety Factors and the Probability of Failure in Fatigue

M. P. Bieniek*
Columbia University, New York

AND

J. C. Joanides†
North American Rockwell, Downey, Calif.

Introduction

The purpose of the factors of safety is to assure integrity of a structure in the presence of conditions which cannot be properly assessed by standard methods of structural design. The most important of these conditions are random deviations of the loadings from their design values and random scatter of the strength of materials.

The fact that the two most important design parameters are random variables points out the suitability of probabilistic and statistical concepts in the formulation of the criteria of structural reliability. Using these concepts, a rational measure of safety can be obtained in terms of the probability of survival (or the probability of failure) of a structure.[1] Consequently, the classical factors of safety acquire a definite meaning if they are related to the probabilities of survival of the structure.

The relations between the probability of failure and certain safety factors for the case of static load has been presented in Ref. 2. This Note deals with a similar problem for the case of fatigue failure of a simple structural element, such as a bar or sheet in tension or compression.

Method of Analysis

Consider a structural element subjected to cyclic loading with stress amplitudes $S_1, S_2, \ldots S_i, \ldots S_k$ and with a mean stress S_m. The number of cycles of loading with a stress amplitude S_i will be denoted by $n(S_i)$. The number of cycles to failure under the same stress amplitude S_i and the same mean stress S_m will be denoted by $N(S_i)$. The number of cycles $N(S_i)$ can be determined from constant-amplitude fatigue tests, and it will be assumed here that $N(S_i)$ is the average of a series of tests.

According to the Palmgren-Miner theory,[3] the fatigue failure occurs if the equation

$$\sum_i [n(S_i)/N(S_i)] = 1 \tag{1}$$

is satisfied. Denoting by $n_F(S_i)$ the numbers of cycles which satisfy Eq. (1) we determine the fatigue life (in cycles) under variable amplitude loading as

$$L_F = \sum_i n_F(S_i) \tag{2}$$

Applying now the factor of safety K, we find the safe life L_K as

$$L_K = L_F/K \tag{3}$$

and the corresponding safe numbers of cycles at the stress amplitudes S_i as

$$n_K(S_i) = n_F(S_i)/K \tag{4}$$

In order to determine the probability of failure corresponding to a value of K, i.e., the probability of failure under the cycles $n_K(S_i)$, it is necessary to employ a probabilistic model of the cumulative fatigue damage in an element subjected to variable-amplitude stress cycles. In this Note, the model proposed by E. Parzen[4] will be used. If the numbers of applied cycles of loading $n(S_i)$, at the stress amplitudes S_i, are random variables, the fractional damage D has the mean value and the variance given by

$$E[D] = \sum_i E[n(S_i)]/E[N(S_i)] \tag{5}$$

$$V[D] = \sum_i E[n(S_i)]\, V[N(S_i)]/E^3[N(S_i)] + V\left\{\sum_i n(S_i)/E[N(S_i)]\right\} \tag{6}$$

where $E[\ldots]$ and $V[\ldots]$ denote the mean value and the variance, respectively, of the quantity in the brackets. With the probability of failure defined as

$$P_F = P[D \geq 1] \tag{7}$$

and assuming that the fractional damage D is distributed normally, we find

$$P_F = \frac{1}{(2\pi)^{1/2}} \int_\eta^\infty e^{-y^2/2} dy \tag{8}$$

where

$$\eta = (1 - E[D])/(V[D])^{1/2} \tag{9}$$

In the case of loading with a given (deterministic) number of cycles at the stress levels S_i, equal to, say, $n_K(S_i)$, Eqs. (5) and (6) become

$$E[D] = \sum_i n_K(S_i)/E[N(S_i)] \tag{10}$$

$$V[D] = \sum_i n_K(S_i) V[N(S_i)]/E^3[N(S_i)] \tag{11}$$

The results of Parzen's theory have been compared to the experimental data reported in Ref. 5 for aluminum alloy 7076-T6 subjected to various types of cyclic loading. From the constant-amplitude test data (Table V of Ref. 5) the values of $E[N(S_i)]$ and $V[N(S_i)]$ have been estimated. Then, for variable-amplitude stresses, specified in Table III of Ref. 5, the probabilities of failure have been computed [using Eqs. (8–11)] as functions of the total number of cycles. These theoretical probabilities, together with the observed proportions of failures in the test program, are shown in Fig. 1, for $S_m = 0$ and $S_m = 10$ ksi. The agreement between the theoretical and the experimental results appears to be reasonably good, which justifies the use of Parzen's method, at least in this preliminary work.

Effect of Scatter of the Fatigue Strength

To gain some insight into the effect of scatter of the fatigue strength on the relation between the safety factor and the

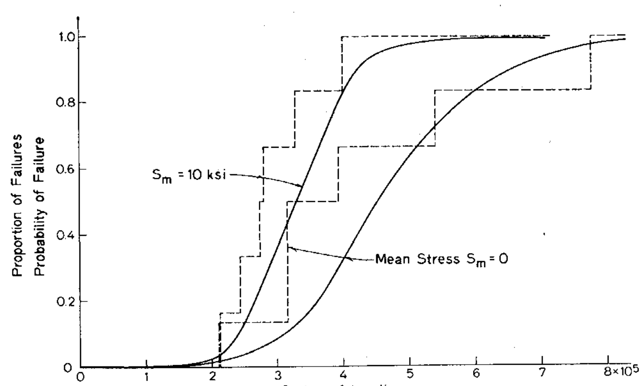

Fig. 1 Theoretical probabilities of failure and test results for 7075-T6 aluminum alloy under variable-amplitude loading with mean stresses 0 and 10 ksi.

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* Professor of Civil Engineering. Member AIAA.

† Member, Structures Staff.

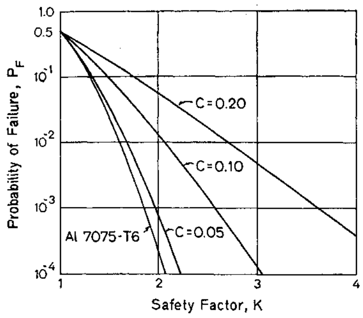

Fig. 2 Probability of failure vs safety factor for three types of scatter of fatigue strength and for 7075-T6 aluminum alloy under variable-amplitude loading.

probability of failure, we shall consider here the case of a deterministic loading for which the number of cycles $n_K(S_i)$ at each stress level is given. In practical analyses, the loading is usually specified in terms of the load spectra. If the number of stress levels is finite, the loading is said to have a discrete spectrum, in which the proportion of cycles with an amplitude S_i is $p(S_i)$. Hence, the total safe number of cycles with the amplitude S_i is

$$n_K(S_i) = L_K p(S_i) \tag{12}$$

If the number of stress levels is very large, a continuous spectrum is used in the form of a function $f(S)$, such that the number of cycles with the amplitudes between, say S_1 and S_2, is

$$n_K = L_K \int_{S_1}^{S_2} f(S)\, dS \tag{13}$$

For computational purposes, a continuous spectrum may be replaced by an equivalent discrete spectrum; for example, with

$$p(S_i) = f(S_i)\Delta S \tag{14}$$

where ΔS is the stress-amplitude interval.

In terms of $p(S_i)$ and L_K, Eqs. (10) and (11) assume the form

$$E[D] = \sum_i L_K p(S_i)/E[N(S_i)] \tag{15}$$

$$V[D] = \sum_i L_K p(S_i)\ V[N(S_i)]/E^2[N(S_i)] \tag{16}$$

Consider a material with the property

$$V[N(S_i)]/E^2[N(S_I)] = \text{const} = C \tag{17}$$

i.e., the scatter of the fatigue life is the same for every stress amplitude. In this case, from Eqs. (10) and (11) or (15) and (16),

$$E(D) = 1/K, \quad V[D] = C/K \tag{18}$$

and the probability of failure follows from Eq. (8). Figure 2 shows the relations between the probability of failure P_F and the safety factor K for three values of C.

An actual material will not, as a rule, exhibit the same scatter for every stress amplitude. Consequently, the values of $E(D)$ and $V(D)$ must be numerically evaluated from Eqs. (15) and (16), as the simple expressions (18) are no longer valid. A computation of this type has been performed for 7076-T6 aluminum alloy tested in Ref. 5, and for the load spectra given in Table III of Ref. 5, with $S_m = 0$ and $S_m =$ 10ksi. The results are given in Fig. 2. The lines corresponding to $S_m = 0$ and $S_m = 10$ ksi are very close; this may be explained by the fact that in both cases the same "gust frequency curve" has been used to establish the load spectrum.

Conclusions

The relation between the safety factor K and the probability of failure in fatigue P_F has been established with the aid of a probabilistic model of fatigue failure under variable-amplitude loading. This relation appears to be of definite significance in selecting rational values of the safety factor. Also, it may be of some help in planning the inspection, maintenance, and replacement for a fleet of aircraft. Further research is needed in the area of the probabilistic models of fatigue failure and their application to more complex, especially multiple-path, structures.

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An Experimental Method for Determining the Condensed Phase Heat of Reaction of Double-Base Propellants

C. E. Kirby*
NASA Langley Research Center, Hampton, Va.

AND

N. P. Suh†
Massachusetts Institute of Technology, Cambridge, Mass.

Nomenclature

c_p = specific heat
$\bar{c}_p$ = average specific heat of gas mixture
D = mass diffusivity
E = activation energy
G = mass flux fraction
Δh_f = standard heat of formation
K = pre-exponential factor in Arrhenius expression
M = total mass flux
Q_r = condensed phase heat of reaction
R = gas constant
T = temperature
W = net rate of loss due to chemical change
x = space coordinate
Y = mass concentration fraction
λ = thermal conductivity
ρ = mass density

Introduction

At present, no acceptable means of measuring the condensed phase heat of reaction of solid propellants exists. Some investigators have attempted to use differential scanning

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* Aerospace Engineer, Flight Vehicles and Systems Division. Member AIAA.

† Associate Professor of Mechanical Engineering.